

False Discovery Rate Based Distributed Detection in the Presence of Byzantines

Aditya Vempaty*, *Student Member, IEEE*, Priyadip Ray, *Member, IEEE*,
Pramod K. Varshney, *Fellow, IEEE*

Abstract

Recent literature has shown that the control of False Discovery Rate (FDR) for distributed detection in wireless sensor networks (WSNs) can provide substantial improvement in detection performance over conventional design methodologies. In this paper, we further investigate system design issues in FDR based distributed detection. We demonstrate that improved system design may be achieved by employing the Kolmogorov-Smirnov distance metric instead of the deflection coefficient, as originally proposed in [1]. We also analyze the performance of FDR based distributed detection in the presence of Byzantines. Byzantines are malicious sensors which send falsified information to the Fusion Center (FC) to deteriorate system performance. We provide analytical and simulation results on the global detection probability as a function of the fraction of Byzantines in the network. It is observed that the detection performance degrades considerably when the fraction of Byzantines is large. Hence, we propose an adaptive algorithm at the FC which learns the Byzantines' behavior over time and changes the FDR parameter to overcome the loss in detection performance. Detailed simulation results are provided to demonstrate the robustness of the proposed adaptive algorithm to Byzantine attacks in WSNs.

Index Terms

False Discovery Rate, Byzantine Attacks, Distributed Detection, Wireless Sensor Networks, Decision Fusion

A. Vempaty and P. K. Varshney are with Department of EECS, Syracuse University, Syracuse, NY 13244. P. Ray is with G. S. Sanyal School of Telecommunications, Indian Institute of Technology, Kharagpur, West Bengal, India. Email: {avempaty, varshney}@syr.edu, priyadip.ray@gssst.iitkgp.ernet.in

I. INTRODUCTION

In recent years, wireless sensor networks (WSNs) have been extensively employed to monitor a region of interest (ROI) for reliable detection/estimation/tracking of events [2] [3] [4] [5]. In this paper, we focus on distributed target detection in WSNs, which has been a very active area of research in the recent past. In distributed detection [6], due to power and bandwidth constraints, each sensor, instead of sending their raw data, sends quantized data (local decision) to a central observer or Fusion Center (FC). The FC combines these local decisions based on a fusion rule to come up with a global decision. There has been extensive research on distributed detection with fusion of local decisions [6]. Optimum fusion rules have been derived for the distributed detection problem under various assumptions [7] [8] [9] [10] [11]. Most of these fusion rules require complete knowledge of the local sensor performance metrics, such as the probability of detection and false alarm. However, in large wireless sensor networks and under complex target signal models, the local sensor performance metrics may not be known or may be very difficult to estimate. To address the scenario of unknown local sensor performance metrics, in [12] [13], the authors have proposed employing the total number of detections (also referred to as the “*count statistic*”) as a decision statistic at the FC. The fusion rule based on the count statistic leads to a decision rule where the sensors decisions are weighed equally, even though the SNR at each sensor may be different.

Under fairly general conditions, obtaining the optimal local decision rules have been shown to be a very difficult problem [6] [14]. Under the conditional independence assumption, it has been shown that identical local quantizers is optimal under asymptotic conditions (i.e., the number of sensors $N \rightarrow \infty$) [15]. Although the optimality of identical quantizers does not hold in general [15] [6] [14], design of non-identical quantizers is computationally very complex and researchers have generally employed identical quantizers based on asymptotic optimality of identical quantizers. Recently, False Discovery Rate (FDR) based distributed detection has been proposed by Ermis and Saligrama [16] and Ray and Varshney [17] [1]. In [1], the authors have proposed employing non-identical thresholds for distributed detection in WSNs based on the control of FDR. It has been shown that under the assumption that the FC employs a test statistic which is linear in count (count here refers to the total number of detections) to reach the global decision, control of the FDR leads to non-identical local decision rules. This scheme provides significant improvement in the global detection performance [1]. In [1], the authors suggest maximization of the deflection coefficient to obtain the FDR design parameter. However, as the count statistic is non-Gaussian in general, maximization of the deflection coefficient does not guarantee optimal global performance [18]. In this paper, we further analyze system design for FDR based distributed detection and demonstrate that

improved system design may be achieved via optimization of the Kolmogorov-Smirnov distance instead of the deflection coefficient.

Most of the research in the field of distributed detection has been carried out under the assumption of a secure network. Only in the recent past, researchers have investigated the problem of security threats [19] on sensor networks. In this paper, we focus on one particular class of security attacks, known as the Byzantine attack [20] [21] [22] [23] (also referred to as the Data Falsification Attack). Byzantine attack involves malicious sensors within the network which send false information to the FC, to disrupt the global decision making process. Byzantines intend to deteriorate the detection performance of the network by suitably modifying their decisions before transmission to the FC. Marano et al. [20] first considered the distributed detection problem in the presence of Byzantines under the assumption that the Byzantines have perfect knowledge of the underlying true hypothesis. In [20], the authors also presented the optimal attacking distributions for the Byzantines such that the detection error exponent is minimized at the FC. Rawat et al. in [21] considered the more practical case when the Byzantines did not have the knowledge of the underlying true hypothesis and also proposed a simple algorithm at the FC to identify the Byzantines in the network. In [24], stochastic resonance [25] [26] was employed to mitigate the effect of Byzantines in a distributed inference network. In our previous work [23], we have analyzed localization in WSNs in the presence of Byzantines and proposed mitigation techniques to make the Byzantines ‘ineffective’. In this paper, we study the performance of the FDR based distributed detection framework in the presence of Byzantines. It is observed that the global detection performance deteriorates rapidly when the fraction of Byzantine sensors increases. Hence, we propose a novel algorithm at the FC, based on a modified Kolmogorov goodness-of-fit test, which detects the fraction of Byzantines present in the network and adaptively changes the FDR parameter to improve the detection performance.

The key contributions of this paper are summarized as follows:

- We propose maximization of the Kolmogorov-Smirnov distance instead of the deflection coefficient to obtain the FDR design parameter and demonstrate that it considerably improves system performance.
- We define a Byzantine attack model and show that the FDR value is controlled even in the presence of Byzantines; however the local sensor detection performance deteriorates considerably when the fraction of Byzantines is large.
- We next study the performance of FDR based distributed detection in the presence of Byzantine attacks and provide analytical and simulation results on the effect of Byzantines on global detection performance.
- Finally, we propose an algorithm which adaptively changes the system parameters by learning the

Byzantines' behavior over time and demonstrate that the proposed algorithm provides improved system performance in the presence of Byzantines.

The remainder of the paper is organized as follows: In Section II, we introduce the system model and lay out the assumptions made in the paper. We also formally define False Discovery Rate (FDR) and briefly discuss the FDR based distributed detection scheme proposed in [1]. We propose some changes to the system design algorithm proposed in [1] and show the improvement in system performance in Section III. In Section IV, we show the performance degradation of FDR based schemes in the presence of Byzantines. We show that although the FDR value is maintained at the specified value, the power of the test reduces. We provide analytical results on the performance of FDR based distributed detection in the presence of Byzantines in Section V. We propose an approximation to the optimal parameter design approach which is computationally efficient and building on it, propose the adaptive distributed detection scheme in Section VI. We conclude with a discussion on possible future directions in Section VII.

II. PRELIMINARIES

A. System Model

We consider a parallel fusion network where N sensors are randomly deployed in the Region of Interest (ROI). Each sensor receives noisy target signals, s_i (for $i = 1, 2, \dots, N$) and makes a decision b_i regarding the presence/absence of the target, which is then transmitted to the FC. The FC makes a global decision ($b_0 \in \{1, 0\}$) regarding the presence/absence of the target using the transmitted local decisions $\{b_i\}_{i=1, \dots, N}$. We assume that the channels between the local sensors and the FC are ideal (for results on distributed detection with imperfect channels, see [27], [28], and references therein). We consider the presence of $M = \alpha N$ ($0 < \alpha < 1$) Byzantines in the network. These Byzantines' aim is to send falsified information to the FC and deteriorate the detection performance. Their model and attack strategy would be described later in Section IV.

As discussed earlier, due to the bandwidth and energy constraints, each local sensor sends a binary decision (0/1) to the FC based on a local hypothesis test. The local sensor's hypothesis test can be formulated as follows:

$$H_0 : s_i = n_i : \text{Target absent} \quad (1)$$

$$H_1 : s_i = a_i + n_i : \text{Target present} \quad (2)$$

where $a_i = \sqrt{P_i}$ is the signal amplitude received at the i^{th} sensor due to the presence of the target and $n_i \in \mathcal{N}(0, 1)$ represent i.i.d. Gaussian noise. In this paper, we assume that the signal power received due

to that emitted by the target drops to zero outside its finite radius of influence (d_0). The general signal model used is

$$P_i = g(d_i) \quad (3)$$

where P_i is the signal power received at the i^{th} sensor which is at a distance d_i from the target. As discussed above, the following $g(\cdot)$ has been used in this paper:

$$g(x) = \begin{cases} P_0, & \text{if } 0 \leq x \leq d_0 \\ 0, & \text{if } x > d_0 \end{cases} \quad (4)$$

The above model is adopted primarily for analytical convenience; however the results provided in this paper may be easily extended to more complex target signal models, such as where the target signal decays exponentially or in an inverse square manner with distance.

The FC makes a global decision based on the vector of local decisions $\underline{b} = (b_1, \dots, b_N)$ received from all the sensors. The binary hypothesis problem at the FC is

$$G_0 : P(\underline{b}; G_0) : \text{Target absent} \quad (5)$$

$$G_1 : P(\underline{b}; G_1) : \text{Target present} \quad (6)$$

where, $P(\underline{b}; G_0)$ and $P(\underline{b}; G_1)$ are the distributions of \underline{b} in the absence of the target (G_0) in the ROI and in the presence of the target (G_1) in the ROI respectively.

Conventionally, for the distributed detection problem, identical decision rules are used at the local sensors. However in [1], FDR based non-identical sensor decision rules have been proposed and shown to be superior to identical decision rules. In the next subsections, we introduce the concept of FDR and provide a brief description of FDR based distributed detection.

B. False Discovery Rate (FDR)

In statistical hypothesis testing, when a family of tests (for e.g., multiple binary hypothesis tests) are conducted, it is often meaningful to define an error rate for the family of tests instead for an individual test. Family wide error rate (FWER) [29] is perhaps the most popular error rate used in the literature. It is defined as the probability of committing any type I error or false alarm. If the error rate for each test is β then the FWER β_F for k tests is

$$\beta_F = P(F \geq 1) = 1 - (1 - \beta)^k \quad (7)$$

where F is the total number of false alarms. As can be seen from (7), as the number of tests k increases, β remains constant but β_F increases. This is a fundamental problem in Multiple Comparison Procedures

(MCP) and classical comparison procedures aim to control this error measure. Bonferroni procedure [30] is a widely employed procedure to control the FWER at a desired rate, but it results in significantly reduced power (probability of detection). A radically different and more liberal approach proposed by Benjamini and Hochberg [31] controls the FDR, defined as the fraction of false rejections among those hypotheses rejected. Formally, FDR is defined as the expected ratio of the number of false alarms (declared H_1 when H_0 is true) to the total number of detections (H_1 declarations consisting of both true and false detections).

TABLE I: Notations to define FDR

	Declared H_0	Declared H_1	Total
H_0 true	W	F	N_0
H_1 true	T	S	$N - N_0$
Total	$N - R$	R	N

From Table I, the ratio of false alarms to the total number of detections can be viewed as the random variable,

$$Q = \begin{cases} \frac{F}{F+S} & \text{if } F + S \neq 0 \\ 0 & \text{if } F + S = 0 \end{cases} \quad (8)$$

FDR (Q_e) is defined to be the expectation of Q ,

$$Q_e = E(Q) \quad (9)$$

This metric was proposed by Benjamini and Hochberg [31] along with the following centralized algorithm to control FDR for multiple comparisons.

Algorithm to control FDR: Suppose p_1, p_2, \dots, p_N are the p-values for N tests and $p_{(1)}, p_{(2)}, \dots, p_{(N)}$ denote the ordered p-values. The p-value for an observation s_i is defined as

$$p_i = \int_{s_i}^{\infty} f_0(s) ds \quad (10)$$

where, $f_0(s)$ is the probability density function of the observation under H_0 . The algorithm by Benjamini and Hochberg [31] which keeps the FDR below a value γ , is provided below.

1. Calculate the p-values of all the observations and arrange them in ascending order.

2. Let d be the largest i for which $p_{(i)} \leq i\gamma/N$.
3. Declare all observations corresponding to $p_{(i)}$, $i = 1, \dots, d$, as H_1 .

Under the assumption of independence of test statistics corresponding to the true null hypotheses (H_0), this procedure controls the FDR at γ . Note that the FDR based decision making system looks for the largest index $i = d$ such that $p_{(d)} \leq d\gamma/N$. There may be other indices $i = l$, where $l < d$ for which the condition $p_{(l)} \leq l\gamma/N$ may be true, but the FDR-based decision system looks for the largest value of i for which this is true. The reason behind this, as discussed in [31] and subsequently pointed out in [1], is to achieve the largest probability of detection while constraining the FDR to less than or equal to γ . Further discussion including the proof of this algorithm is omitted for the brevity of the paper and may be found in [31].

The above algorithm requires the ordering of p-values and the procedure conventionally needs centralized processing. For the distributed detection problem considered in this paper, the sensors can only send one bit to the FC and hence a distributed ordering scheme is necessary. A decentralized FDR procedure has been proposed in [1] that achieves the same performance as the Benjamini and Hochberg procedure. This algorithm is based on the fact that the only information required by the FC is the number of H_1 declarations (henceforth referred to as the count statistic, Δ). The algorithm and further discussion may be found in [1].

III. SYSTEM DESIGN

In this section, we summarize the design guidelines for FDR based distributed detection proposed in [1] and show that the system design aspects need to be re-visited. The number of design parameters in a FDR based distributed detection system [1] are γ and T_{FDR} , where γ is the FDR parameter and T_{FDR} is the global threshold. For the sake of comparison, we also study system design for an identical threshold scheme, where the design parameters are τ and T_{IT} , where τ is the local observation threshold parameter ($Q(\tau) = p_{fa}$ is the threshold on the p-values) and T_{IT} is the global threshold. The system-wide probability of false alarm for the FDR based distributed detection system is given by,

$$P_{FA} = P(\Delta > T_{FDR}; G_0) + \kappa P(\Delta = T_{FDR}; G_0) \quad (11)$$

where κ is the randomization parameter. Similarly, the system-wide probability of detection is given by

$$P_D = P(\Delta > T_{FDR}; G_1) + \kappa P(\Delta = T_{FDR}; G_1) \quad (12)$$

where the p.m.f of the count statistic is given by Propositions 2, 4, 5 and 6 of [1]. The system-wide performance metrics for the identical threshold scheme may be obtained similarly.

It has been shown in [1] that the choice of optimal FDR parameter γ or identical decision threshold τ , where optimality is with respect to system-level detection performance is a difficult problem. Optimizing the ROC via simulation or numerical computation is very complex (see [1] for further discussion). A computationally less intensive approach is to use distance measure based optimization for system design. Motivated by this, in [1], optimization of deflection coefficient of the count statistic (Δ) was proposed to find the optimal γ or τ . Intuitively increased deflection coefficient generally implies greater separation between the pmfs of the count statistic under global hypothesis G_0 and G_1 and is likely to lead to better detector design. Also, the distribution of the count statistic under asymptotic conditions is Gaussian for which it is known that maximization of the deflection coefficient leads to an optimal detector [18]. However, in this paper, we show that maximization of the deflection coefficient may not be the best design criterion for FDR based detection system under non-asymptotic conditions. Since the distribution of the count statistic (Δ) is non-Gaussian in general, it is likely that the deflection coefficient fails to characterize its performance completely. We study the performance of several candidate distance measures (such as Kullback-Leibler Divergence, Bhattacharya Distance and Kolmogorov-Smirnov Distance) for system design.

As a motivating example, we perform the following simulation. Let $N = 20$ sensors be randomly distributed within the circular ROI of radius $R = 10$. The pdf of the sensor locations r (r is measured from the target location) is given by $f_R(r) = 2r/R^2, 0 \leq r \leq R$. The optimal values of parameters γ and τ obtained for a target model with power $P_0 = 5$ and radius of influence $d_0 = 5$ depend on the system-wide probability of false alarm, P_{FA} . For $P_{FA} = 0.1$, the optimal parameters that maximize P_D are found to be $\gamma_{opt} = 0.25$ and $\tau_{opt} = Q^{-1}(0.005)$. However, when deflection coefficient is used, we get the optimal parameters as $\gamma_{opt}^d = 0.008$ and $\tau_{opt}^d = Q^{-1}(0.00005)$. The optimization has been done numerically through simulations and the plots have been omitted for the sake of brevity of the paper.

The above example shows that the deflection coefficient does not always yield the optimal value of the parameters and as shown can deviate substantially from the optimal. This makes it necessary to come up with a distance measure which is computationally more efficient than using ROC based optimization, and provides a better approximation to the optimal solution. In this paper, we present some comparative performance results between four candidate measures: Deflection Coefficient [12], Kullback-Leibler Divergence [32], Bhattacharya Distance and Kolmogorov-Smirnov Distance [33]. Table II compares the optimal parameter values found by the optimization of each of the distance measures against the true optimal found by ROC based optimization (P_D maximization while fixing $P_{FA} = 0.1$) for both the schemes: FDR based threshold design and identical threshold design.

TABLE II: Performance Comparison using different distance measures

Metrics	FDR based threshold scheme (γ)	Identical threshold scheme ($Q(\tau) = p_{fa}$)
ROC based optimization	0.25	0.005
Deflection Coefficient optimization	0.008	0.00005
Kullback-Leibler Divergence optimization	0.1	0.00005
Bhattacharya Distance optimization	0.2	0.01
Kolmogorov-Smirnov Distance optimization	0.2	0.005

For the example considered, as can be seen from Table II, the Kolmogorov-Smirnov Distance (KSD) best approximates the optimal parameters compared to other candidate measures for both the FDR based scheme and the identical threshold scheme. We also noted a similar result for a wide range of simulation parameters. The deflection coefficient fails to find the optimal parameters as it only looks at the first and the second moments of the distributions which is not sufficient when the underlying distribution is non-Gaussian. In our context, the count statistic (Δ) is non-Gaussian and is a discrete RV and, therefore, deflection coefficient does not characterize the distribution of count statistic. In the remainder of this section, we formally define the Kolmogorov-Smirnov distance and perform some empirical studies based on K-S distance to find the optimal parameter values for distributed detection schemes (both FDR based scheme and identical threshold scheme).

Definition The Kolmogorov-Smirnov distance (K-S distance) is defined as the maximum value of the absolute difference between two cumulative distribution functions [33].

The Kolmogorov Smirnov measures the distance between the empirical distribution function of the sample data and the cumulative distribution function of the reference distribution. The K-S distance is non-parametric and distribution free, i.e., it makes no assumption about the underlying data distribution. The K-S distance between two cdfs $F(\cdot)$ and $G(\cdot)$ is defined as

$$KSD(F(\cdot), G(\cdot)) = \sup_{0 \leq x \leq 1} |F(x) - G(x)| \quad (13)$$

In our context, $F(\cdot)$ and $G(\cdot)$ represent the cdfs of the count statistic (Δ) under global hypotheses G_0 and G_1 . The optimal parameter value of the local decision threshold parameter (γ for FDR based scheme or p_{fa} for identical threshold scheme) is found by maximizing the K-S distance.

It is important to note here that the optimal parameter value depends on the system requirement of system-wide probability of false alarm (P_{FA}). When $P_{FA} = 0.2$ is used, the optimal parameter values based on ROC optimization come out to be $\gamma = 0.2$ and $p_{fa} = 0.01$ which again are close to the optimal parameter values found by using the K-S distance measure for optimization. It is important to note here that although the optimal parameter value changes, the FDR based decision threshold scheme proposed in [1] outperforms the identical threshold scheme even when K-S distance is used as the distance metric as seen in Fig. 1. Fig. 1 shows the ROC obtained by using the parameters obtained by K-S distance optimization instead of deflection coefficient optimization. Note that the ROCs obtained by deflection coefficient optimization in [1] are not as good as the ROCs obtained in Fig. 1 using K-S distance optimization.

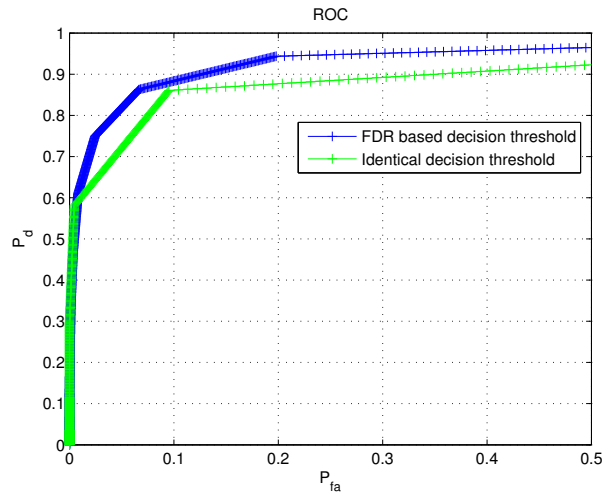


Fig. 1: ROC using the optimal parameter values found using K-S distance

From empirical studies, K-S distance seems to provide a good approximation to the optimal. We hypothesize that since K-S distance measures the overlap between the distributions and is non-parametric in nature, it provides better results compared to deflection coefficient maximization. However, to find the best distance measure to use is an interesting but difficult problem to solve considering the large number of candidate measures available in the literature (Ali-Silvey Distance Measures [34]). We leave this for future work and for the remaining part of the paper, we use K-S distance or ROC based optimization to find the optimal local threshold parameters.

IV. CONTROL OF FDR IN THE PRESENCE OF BYZANTINES

In this section, we consider the problem of FDR based distributed detection in the presence of Byzantines. We first show the effect of Byzantines on the control of FDR in multiple comparison problems and in subsequent sections analyze the effect of Byzantines on FDR based distributed detection.

Byzantines are the local sensors which send falsified information to the FC to deteriorate system performance. Since control of FDR is based on p-values, the Byzantines' attack strategy would be to report a falsified p-value denoted by $q_i = h(p_i)$, where $h(\cdot)$ is a transformation used by the Byzantines and p_i is the true p-value of the i^{th} sensor. The transformation $h(\cdot)$ needs to satisfy the properties listed below:

1. $h(\cdot)$ should be a function whose domain and range are $[0, 1]$ or $h : [0, 1] \rightarrow [0, 1]$
2. $h(\cdot)$ should be a decreasing function. This property is essential since the Byzantines' aim is to deteriorate the detection performance [20]. As the p-value represents the 'confidence' of the target being absent, they would like to report falsified information by reversing it. A decreasing function ensures a higher q-value for a lower p-value and vice-versa.

One possible transformation is the linearly decreasing function $h(p) = 1 - p$, which is equivalent to flipping of the local decision. This transformation has been shown to be the optimal attack for Byzantines in distributed detection using identical local thresholds [21] and target localization with quantized data [23]. In the remainder of this paper, we use the above transformation to model the Byzantine attack. Therefore, if p_i represents the true p-value of the i^{th} sensor, then the reported p-values are given by

$$q_i = \begin{cases} p_i, & \text{if } i^{th} \text{ sensor is honest} \\ 1 - p_i, & \text{if } i^{th} \text{ sensor is Byzantine} \end{cases} \quad (14)$$

In the rest of the section, we show the effect of Byzantines on the control of FDR. As mentioned previously, an important aspect of the FDR based detection is the control of FDR value at the pre-determined threshold γ . The FDR control algorithm provided earlier in Section II-B, controls the FDR value at γ when the true Hypothesis is H_0 and at a value less than or equal to γ when the true Hypothesis is H_1 . We now prove that the FDR value is controlled even in the presence of Byzantines.

Proposition 4.1: Let N sensors be randomly deployed in the ROI. The local sensors report local decisions to the FC using their p-values and the FC makes the final decision regarding the presence/absence of the target using these local decisions. Let there be M Byzantines in the network which transform the p-values according to (14) and report decisions based on the falsified p-values. For independent local

sensors under global hypothesis G_0 , the FDR control algorithm proposed in [31] control the FDR value at the pre-determined threshold γ , even in the presence of Byzantines.

Proof: In order to prove this proposition, we need to find the distribution of p-values under both the hypotheses in the presence of Byzantines. The Byzantines' model used in this paper is that of transforming the p-values according to (14). For a Gaussian random variable $N(\phi, 1)$, the density of the p-value $f_\phi(u)$ is given by [1]

$$f_\phi(u) = \exp\left(-\frac{\phi^2}{2}\right) \exp(\phi Q^{-1}(u)), \quad 0 \leq u \leq 1 \quad (15)$$

From this result, under null hypothesis H_0 , the true p-values follow uniform distribution ($\phi = 0$). Since the Byzantines' effect is a transformation given by (14), the distribution of the reported p-values under H_0 can be found by the transformation of random variables $V = 1 - U$. Due to probability netgral transform, the reported p-values also follow uniform distribution under null hypothesis.

The true p-value under the alternate hypothesis H_1 follows the distribution given by (15) with $\phi = \sqrt{(P_0)}$. For the Byzantines, the reported p-value is given by $q = 1 - p$. Using the change of variables, $V = 1 - U$,

$$f_\phi(v) = f_\phi(u)|_{u=1-v} \left| \frac{du}{dv} \right| \quad (16)$$

This gives us the distribution of the reported p-values $f_V(v)$ as

$$f_V^B(v) = \exp\left(-\frac{\phi^2}{2}\right) \exp(\phi Q^{-1}(1 - v)) \quad \text{for } 0 \leq v \leq 1 \quad (17)$$

where ϕ is the received signal amplitude at the local sensor which is either 0 or $\sqrt{(P_0)}$ under hypothesis H_0 and H_1 respectively.

The proof of this proposition then follows from the straightforward observation that the FDR algorithm proposed by Benjamini and Hochberg [31] controls the FDR value for any configuration of the false null hypothesis. The condition that p-values are uniformly distributed under true null hypothesis is still satisfied and since the proof does not depend on the p-values' distribution under H_1 , the FDR value is controlled at the pre-determined threshold. ■

We now show via simulations that the FDR value is controlled at γ even in the presence of Byzantines. Figs. 2 and 3 show the FDR value with varying fraction of Byzantines (α) present in the network. Let us consider a distributed detection system with the parameters: $N = 20$, $R = 10$, $d_0 = 5$, $P_0 = 5$ and the FDR parameter $\gamma = 0.25$. The simulation results are for 5×10^4 Monte-Carlo runs. As can be seen from the figures, the FDR value is maintained at γ under G_0 and at a value less than or equal to γ under G_1 even in the presence of Byzantines.

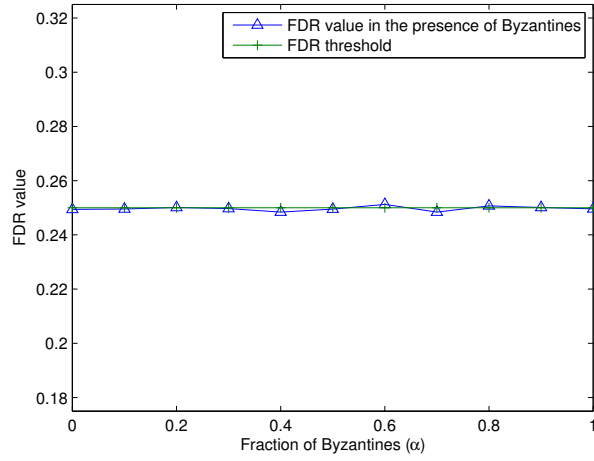


Fig. 2: FDR value against the fraction of Byzantines when the true hypothesis is G_0

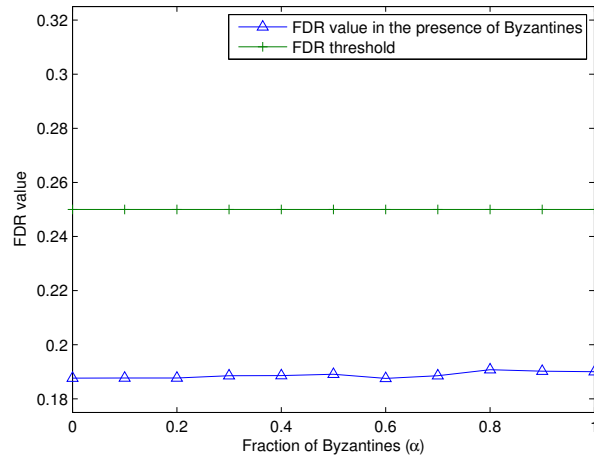


Fig. 3: FDR value against the fraction of Byzantines when the true hypothesis is G_1

This ineffectiveness of Byzantines on the FDR value is expected because the effect of Byzantines changes the order of the reported p-values (q-values) and the largest threshold crossing index would be different as compared to the largest threshold crossing index on the original p-values. In other words, in the presence of a target, most of the true p-values are small and therefore the threshold crossing would be closer to the right extremal resulting in a high number of detections as depicted by an example in Fig. 4a. In the presence of Byzantines, the p-values get transformed as defined by (14) and the reported p-values become larger and the threshold crossing shifts to the left, as shown in Fig. 4b. This reduces the number

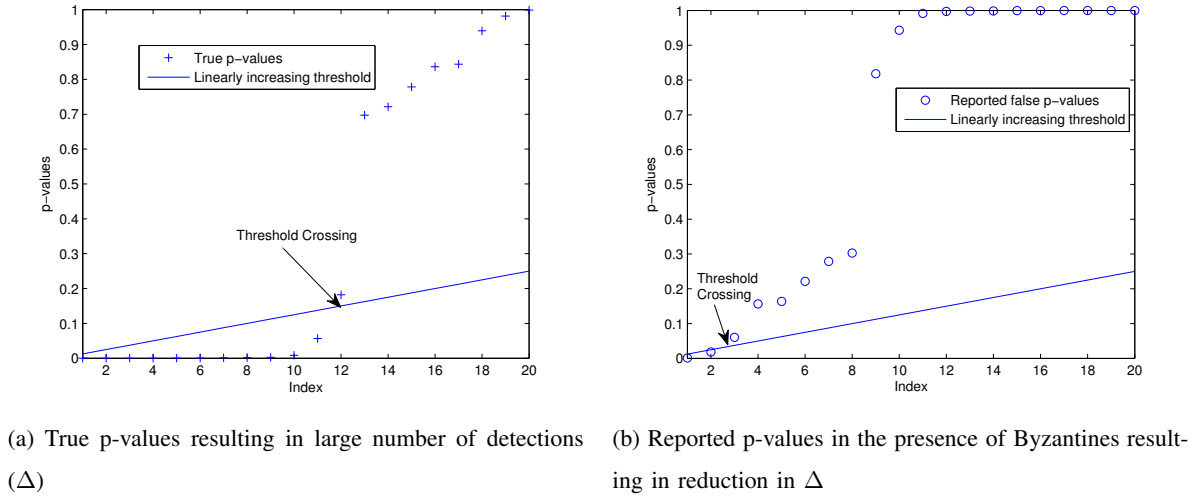


Fig. 4: p-values against linearly increasing threshold in the presence of target

of detections and it is equivalent to looking at an earlier threshold crossing index on the true p-values. As pointed out in Section II-B, the FDR algorithm looks at the largest index satisfying $p_{(i)} \leq i\gamma/N$ to maximize the power of the test. Observe that when $\alpha = 1$, i.e., all the sensors are Byzantines, the order of the q-values is reversed as compared to the p-values and the FDR algorithm based on the q-values ends up looking at the smallest index of p-values satisfying $p_{(i)} \leq i\gamma/N$ rather than the largest index. Under this observation, we conjecture that for $0 < \alpha < 1$, the FDR algorithm based on the q-values ends up looking at an index of p-value between the largest and the smallest indices satisfying $p_{(i)} \leq i\gamma/N$. Since, the proof of control of FDR does not depend on whether it is the largest threshold crossing index or not, the FDR value is maintained at the required threshold. However, the power of the test degrades in the presence of Byzantines.

In the above discussion, we have shown that the FDR value is not affected by the presence of Byzantines. However, it was also pointed out that the FDR control algorithm for the reported q-values is equivalent to looking at an earlier index on the true p-values. In the presence of Byzantines, the number of detections is reduced and, therefore, the distribution of the count statistic (number of detections) under G_1 is now closer to the distribution of count under G_0 which remains unchanged in the presence of Byzantines. This makes it difficult to distinguish between the two hypotheses.

In the following section, we explore this intuitive observation that the Byzantines bring the distribution of the count statistic under G_0 closer to its distribution under G_1 in the context of FDR based distributed

detection. We show analytically the effect of Byzantines on the count statistic and derive the distributions of the count statistic under G_0 and G_1 .

V. FDR BASED DISTRIBUTED DETECTION IN THE PRESENCE OF BYZANTINES

In order to understand the behavior of FDR based distributed detection scheme in the presence of Byzantines, we require the knowledge regarding the p.m.f. of the count statistic (Δ) in the presence of Byzantines. These results are provided in the rest of this section.

A. Probability mass function of Count (Δ)

Let the observed p-values be denoted by the random variables $\{U_i\}_{i=1,\dots,N}$ and the reported p-values (q-values) be denoted by the transformed random variables $\{V_i\}_{i=1,\dots,N}$ where the transformation is as follows

$$V_i = \begin{cases} U_i & \text{if } i \text{ is an honest sensor} \\ 1 - U_i & \text{if } i \text{ is a Byzantine sensor} \end{cases} \quad (18)$$

Proposition 5.1: The probability of $\Delta = i$ local false alarms (count) for N sensors containing $M = \alpha N$ Byzantines in the ROI, and control of FDR at γ under G_0 (absence of target in the ROI) is given by

$$P(\Delta = i; G_0) = \binom{N}{i} (1 - \gamma) \left(\frac{i\gamma}{N}\right)^i \left(1 - \frac{i\gamma}{N}\right)^{N-i-1} \quad (19)$$

In the absence of a target, the p-values of both the Byzantines and the honest sensors are uniformly distributed, that is both U_i and V_i are uniformly distributed. Therefore, the result remains the same as derived by Finner and Roters [35] using Dempster's formula for barrier crossing for uniform random variables, irrespective of the presence of Byzantines. Similarly, the asymptotic distribution can be found as

$$\lim_{N \rightarrow \infty} P(\Delta = i; G_0) = \frac{i^i}{i!} (1 - \gamma) \gamma^i \exp(-i\gamma) \quad (20)$$

Proposition 5.2: The probability of $\Delta = i$ local detections (count) for N sensors containing $M = \alpha N$ Byzantine sensors in the ROI, and control of FDR at γ under G_1 (presence of target in ROI) is given by

$$P(\Delta = i; G_1) = \frac{1}{\binom{N}{M}} \sum \int_{v_{N,N}=\gamma}^1 \cdots \int_{v_{i+1,N}=\left(\frac{(i+1)}{N}\gamma\right)}^{v_{i+2,N}} \int_{v_{i,N}=0}^{i\gamma/N} \cdots \int_{v_{1,N}=0}^{v_{2,N}} N! f_{V_1}(v_1) f_{V_2}(v_2) \cdots f_{V_N}(v_N) dv_{1,N} dv_{2,N} \cdots dv_{N,N} \quad (21)$$

Also, asymptotically, i.e., for large N ,

$$P(\Delta = i; G_1) = \sum_{k=\max(0, M-N+i)}^{\min(M, i)} \binom{M}{k} \binom{N-M}{i-k} (\bar{p}_D^B)^k (1 - \bar{p}_D^B)^{M-k} (\bar{p}_D^H)^{i-k} (1 - \bar{p}_D^H)^{(N-M)-(i-k)} \quad (22)$$

or

$$P(\Delta = i; G_1) = \binom{N}{i} (\bar{p}_D)^i (1 - \bar{p}_D)^{N-i} \quad (23)$$

where \bar{p}_D^H is the average probability of reporting ‘1’ under G_1 for an honest local sensor, \bar{p}_D^B is the average probability of reporting ‘1’ under G_1 for a Byzantine local sensor and $\bar{p}_D = \alpha \bar{p}_D^B + (1 - \alpha) \bar{p}_D^H$ is the average probability of reporting ‘1’ under G_1 .

Proof: The proof is provided in Appendix A ■

It is interesting to observe here that the Byzantines only affect the p.m.f of the count statistic (Δ) under G_1 , while the p.m.f under G_0 remains the same. The reason behind this is that the p-values under G_0 are uniformly distributed and a transformation $h(p) = 1 - p = q$ still keeps the q-values uniformly distributed under G_0 . However, the p.m.f under G_1 changes as the p-values are no longer uniformly distributed.

B. Numerical Results

In this section, we provide numerical and simulation results to validate the analytical expressions obtained for the p.m.f of the count statistic under FDR based threshold design. Tables III, IV and V show the numerical and simulation results of analytically derived $P(\Delta = i; G_1)$ for FDR based scheme for different values of α . The signal and target parameters are: $P_0 = 3$, $R = 10$, $d_0 = 3$, $N = 4$ and FDR parameter $\gamma = 0.1$. The integrals given in (21) have been evaluated using Monte Carlo integration methods [36]. The simulation results are for 5×10^5 Monte Carlo runs. These tables show that the numerical and simulation results match very closely.

TABLE III: Numerical and Simulation results for $P(\Delta = i; G_1)$ for control of FDR when $\alpha = 0$

Count	0	1	2	3	4
$P(\Delta; G_1)$ (Numerical)	0.7777	0.1777	0.0407	0.0061	3.1544×10^{-4}
$P(\Delta; G_1)$ (Simulations)	0.7756	0.1773	0.0401	0.0065	5.1000×10^{-4}

TABLE IV: Numerical and Simulation results for $P(\Delta = i; G_1)$ for control of FDR when $\alpha = 0.5$

Count	0	1	2	3	4
$P(\Delta; G_1)$ (Numerical)	0.8355	0.1347	0.0227	0.0027	1.8121×10^{-4}
$P(\Delta; G_1)$ (Simulations)	0.8383	0.1353	0.0233	0.0028	1.7000×10^{-4}

TABLE V: Numerical and Simulation results for $P(\Delta = i; G_1)$ for control of FDR when $\alpha = 1$

Count	0	1	2	3	4
$P(\Delta; G_1)$ (Numerical)	0.9022	0.0789	0.0106	0.0012	6.2896×10^{-5}
$P(\Delta; G_1)$ (Simulations)	0.9084	0.0793	0.0111	0.0011	6.4000×10^{-5}

In Figs. 5, 6 and 7, we provide the simulation results and the analytical approximation for the p.m.f of the count statistic $P(\Delta; G_1)$ for a large number of sensors in the ROI. The simulation parameters are $N = 500$, $P_0 = 15$, $R = 10$, $d_0 = 3$ and FDR parameter $\gamma = 0.0077$. The simulation results are for 5×10^4 Monte Carlo runs. The simulation results demonstrate that the asymptotic expressions for $P(\Delta; G_1)$ match the simulation results very well.

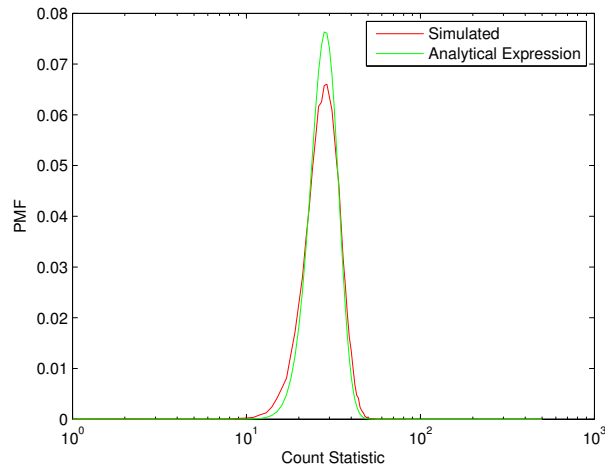


Fig. 5: Simulated and analytical results for $P(\Delta; G_1)$ for FDR based scheme under asymptotic conditions when $\alpha = 0.1$

Fig. 8 shows the reduction in the detection performance with the increase in number of Byzantines in the network. The simulation parameters are: $R = 10$, $d_0 = 5$, $P_0 = 5$, $N = 20$, system-wide probability

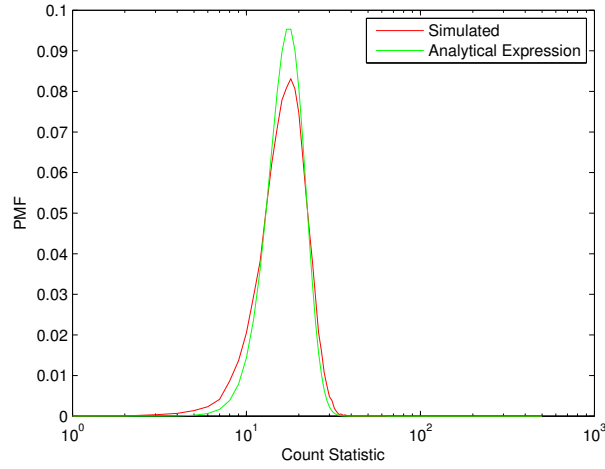


Fig. 6: Simulated and analytical results for $P(\Delta; G_1)$ for FDR based scheme under asymptotic conditions when $\alpha = 0.4$

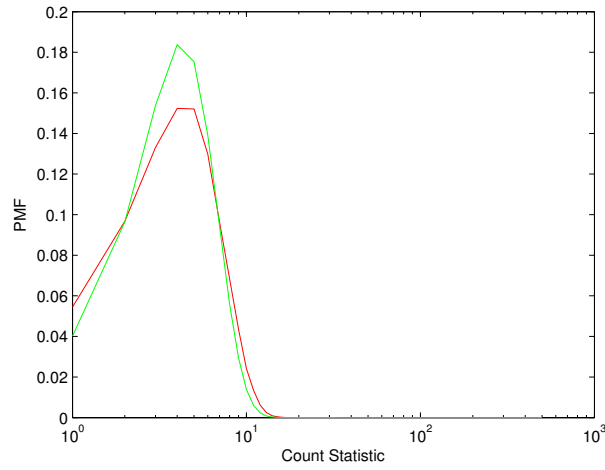


Fig. 7: Simulated and analytical results for $P(\Delta; G_1)$ for FDR based scheme under asymptotic conditions when $\alpha = 0.8$

of false alarm is fixed at $P_{FA} = 0.1$. This yields optimal FDR parameter as $\gamma = 0.25$. The simulation results are for 1×10^4 Monte-Carlo runs. This shows that the Byzantines reduce the power of the test (detection probability) even though the FDR value is maintained at the prescribed threshold. This leads to the interesting problem of obtaining the optimal transformation $h(\cdot)$ for the Byzantines which leads to the worst detection performance at the FC. This analysis is much more complex and will be explored

in the future.

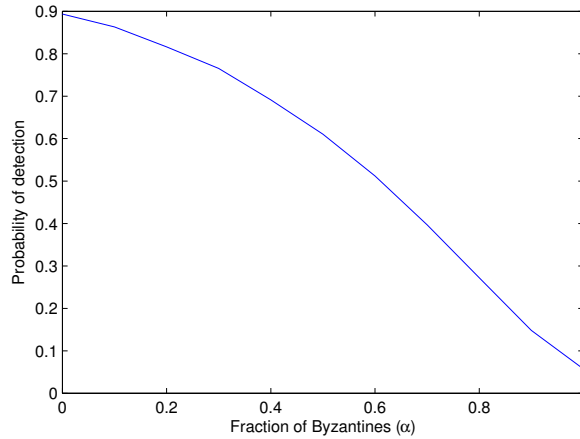


Fig. 8: Probability of detection against the fraction of Byzantines when the true global hypothesis is G_1 for $P_{FA} = 0.1$

VI. ADAPTIVE FDR BASED DISTRIBUTED DETECTION

In this section, we first demonstrate that the optimal design parameter for FDR based scheme depends on the fraction of Byzantines (α). If prior information is available regarding α , we can design our system such that the performance is optimized for the given value of α . However, in a dynamically changing environment, it becomes important that we learn this fraction (α) over time and change our system design parameters adaptively. In this section, we propose an adaptive algorithm which learns the maliciousness of network over time and changes the design parameters to improve the system's detection performance in a dynamic manner. In other words, we learn the effect of the Byzantines on the network and mitigate their effect by adaptively changing the system parameters.

A. Optimal parameter design

In this subsection, we first give design guidelines for FDR based distributed detection in the presence of Byzantines. For a fixed system-wide probability of false alarm given by

$$P_{FA} = P(\Delta > T; G_0) + \kappa P(\Delta = T; G_0) \quad (24)$$

where T is the global threshold for the count statistic used at the FC and κ is the randomization parameter. The optimal local threshold parameter (γ for FDR-based scheme) is found by maximizing the system-wide

probability of detection (P_D). P_D is given by

$$P_D = P(\Delta > T; G_1) + \kappa P(\Delta = T; G_1) \quad (25)$$

where the p.m.f of the count statistic is given by Propositions 5.1 and 5.2. As can be seen from Proposition 5.1, the distribution of count statistic under G_0 does not depend on α and, therefore, the expression of P_{FA} remains the same irrespective of the value of α . In this section, we show through simulations that the optimal parameter for FDR-based approach varies with α . Intuitively, the Byzantines decrease the distance (such as KL divergence or KS distance) between the pmfs of the count under global hypotheses G_0 and G_1 . It is, therefore, important to re-optimize the local threshold parameters to increase the distance between these pmfs as much as possible for this fixed value of α .

For system and target parameters given by $N = 20$, $R = 10$, $P_0 = 5$, $d_0 = 5$ and $P_{FA} = 0.1$, we have shown in Section III that the optimal local threshold parameter is $\gamma = 0.25$ for the FDR based scheme. For the FDR based algorithm, the optimal parameter γ changes with α from $\gamma_{low} = 0.25$ to $\gamma_{high} = 0.1$ as shown in Fig. 9. The simulations were performed for multiple values of α ranging between 0 and 1 but the figures have been omitted for the sake of brevity.

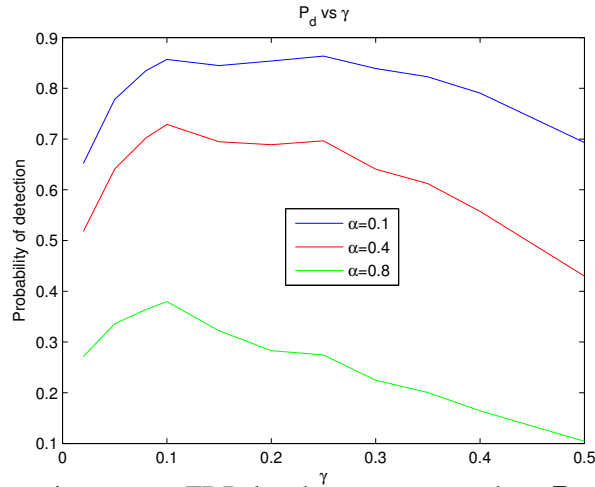


Fig. 9: Probability of detection versus FDR local parameter γ when $P_{FA} = 0.1$ and for varying α

From these extensive simulations it was observed that the optimal parameter remains nearly constant for different intervals of α . For $\alpha \leq 0.2$, $\gamma_{opt} = \gamma_{low} = 0.25$ and for $\alpha > 0.2$, $\gamma_{opt} = \gamma_{high} = 0.1$. Using this information, we can re-simulate Fig. 8 using the optimal parameters. Fig. 10 shows the improvement in detection performance of FDR-based scheme using adaptive optimal parameters.

Since we have shown that the optimal parameter value depends on the fraction of Byzantines present in the network, it becomes important to learn this parameter to adaptively re-design the system using

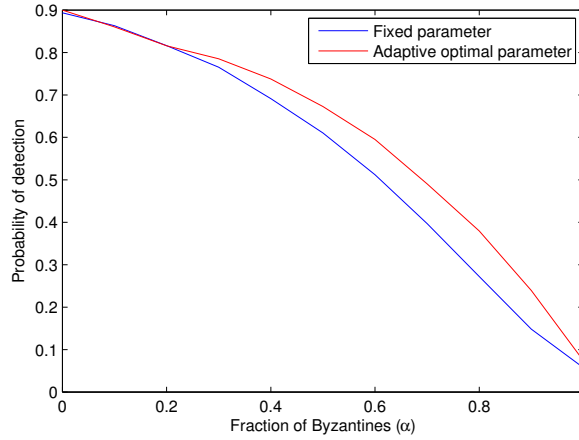


Fig. 10: Probability of detection against the fraction of Byzantines when the true hypothesis is H_1 using optimal parameters

the parameters at hand (local thresholds). We use a modified Kolmogorov-Smirnov Test, proposed in the following sub-section, to learn the fraction of Byzantines present in the network.

B. Modified Kolmogorov-Smirnov Test

Kolmogorov-Smirnov (K-S) test [37] is a goodness-of-fit test which compares a sample observed data with a reference null hypothesis distribution. It quantifies a distance metric (Kolmogorov-Smirnov distance) between the sample empirical c.d.f and the null hypothesis c.d.f to decide the goodness-of-fit. It is typically used only for continuous distribution but Conover [38] has extended this to cover the case of discontinuous distributions. Since, the only information known by the FC at every time instant is the count statistic (Δ), a goodness-of-fit test on the count statistic has to be used to decide the range of α .

Description of the test [38]: Let X_1, X_2, \dots, X_n represent a random sample of size n . Denote the null hypothesis by

$$H_0 : F(x) = H(x) \quad \text{for all } x, \quad (26)$$

where $F(x)$ is the unknown population distribution function, and $H(x)$ is the hypothesized distribution function with all parameters specified. $H(x)$ may be continuous, discrete, or a mixture of the two types. Let $S_n(x)$ represent the empirical distribution function,

$$S_n(x) = \frac{1}{n}(\text{the number of } X_i\text{'s which are } \leq x), \quad \text{for all } x. \quad (27)$$

The algorithm proposed by Conover [38] gives a critical value of the sample which quantifies the confidence of null hypothesis being true. The test statistic D is defined as

$$D = \sup_x |H(x) - S_n(x)| \quad (28)$$

In our case, it is a binary hypothesis test and we would like to compare the sample with the distribution of the count statistic under low α regime and high α regime.

$$K_0 : P(\Delta; G_1) \quad \text{for } \alpha = \alpha_{low} \quad (29)$$

$$K_1 : P(\Delta; G_1) \quad \text{for } \alpha = \alpha_{high} \quad (30)$$

Here, we modify the algorithm provided by Conover [38] to generate test statistics D_i for each of the hypotheses (K_i). Since we have the distribution of the count statistic for both the hypotheses (using the analytical expressions derived in Section V-A), we can find the test statistics under both the hypotheses using the given sample data. We then decide the hypothesis H_i for which the D_i is larger. The advantage of using K-S test is that it performs well even for relatively small number of samples (e.g., 20-30 samples) compared to Pearson's Chi-Square test [39] which requires a larger number of samples.

C. Adaptive Algorithm

In this subsection, we propose an adaptive algorithm based on the modified K-S test described above. In this algorithm, at every time instant t , the FC stores the count value of the previous T_0 time instants for which the global decision of G_1 was made. Using these T_0 data samples, it makes a decision regarding the region in which α lies using the modified K-S test described in Section VI-B. Depending on the decision made, it changes the detector parameters. Let K_i denote the decision made using the modified K-S test, then the detector parameters are changed as

$$K_i = \begin{cases} K_0 : \gamma = \gamma_{low}, T = T_{low}, \kappa = \kappa_{low} \\ K_1 : \gamma = \gamma_{high}, T = T_{high}, \kappa = \kappa_{high} \end{cases} \quad (31)$$

The global threshold parameters (T, κ) also need to be changed in order to maintain the system-wide false alarm probability P_{FA} at the desired value.

We now provide simulation results of our proposed adaptive algorithm. The system and target parameters are: $N = 20$, $R = 10$, $P_0 = 5$, $d_0 = 5$ and $P_{FA} = 0.1$. This gives us the optimal FDR parameters as $\gamma_{low} = 0.25$ and $\gamma_{high} = 0.1$. For the K-S hypothesis test, we have used $T_0 = 30$ samples and the distributions for $\alpha_{low} = 0$ and $\alpha_{high} = 0.5$ under the two hypotheses have been found using (21) given

in Section V-A. The system is initially Byzantine free, i.e., $\alpha = 0$. At $t = 30$, α changes to 0.7. In Fig. 11, the global detection probability is plotted against time for the proposed adaptive algorithm and a non-adaptive algorithm which continues to use the initial detector parameters. As can be observed, the detection performance improves when the adaptive algorithm is used.

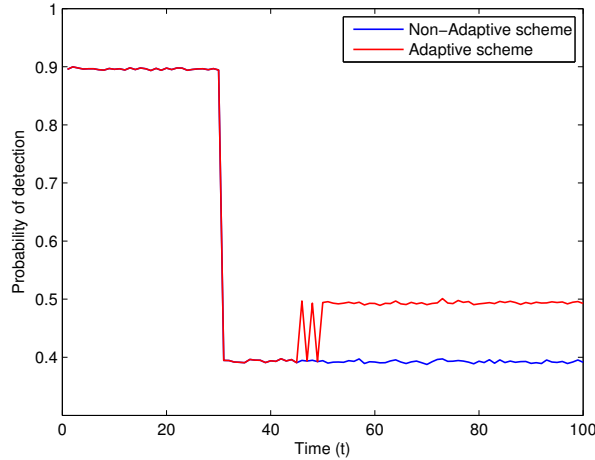


Fig. 11: Probability of detection versus time when α changes from 0 to 0.7 at $t = 30$

VII. CONCLUSION & FUTURE WORK

In this paper, we explored the problem of FDR based distributed detection in the presence of Byzantines. Building on the work of [1], we showed that system performance can be improved in the non-asymptotic cases by the use of Kolmogorov-Smirnov distance as the system design metric instead of deflection coefficient. We explored the system in the presence of Byzantines and showed that the global network performance degrades in the presence of Byzantines although the FDR value is still controlled at the pre-determined threshold (γ). We analyzed the system performance both theoretically and numerically and proposed an adaptive approach to improve the performance which degraded in the presence of Byzantines. The proposed scheme learns the fraction of Byzantines present in the network and adaptively changes the system parameters to improve the global detection performance.

There are several directions for future work on this problem. One could explore other distance measures [34] which characterize the system performance and can be used for system design. The optimal attack strategy for the Byzantines needs to be derived as it would be interesting to see how the performance of the network depends on the optimal attack strategy of the Byzantines defined by $h_{opt}(\cdot)$. Here, we

considered the Neyman-Pearson framework. This work could be extended to a Bayesian framework where the problem is to detect the presence of a random target. For this, one may need to use the Bayesian version of FDR called Bayesian FDR [40] or pFDR [41]. In [40], the rule for control of Bayesian FDR has been proposed which can be used to design local sensor thresholds in distributed detection under Bayesian framework similar to the present work.

APPENDIX A

PROOF OF PROPOSITION 5.2

The pdf of a p-value u of a true sensor observation located at a radial distance r from the target is given by

$$f(u; r) = \exp\left(-\frac{\phi^2}{2}\right) \exp(\phi Q^{-1}(u)), 0 \leq u \leq 1 \quad (32)$$

where $\phi = 0$ if $r > d_0$ and $\phi = \sqrt{P_0}$ if $r \leq d_0$. The marginal pdf of the p-values in the presence of target can be found as

$$f_U(u) = \int_0^{d_0} \exp\left(-\frac{P_0}{2}\right) \exp(\sqrt{P_0} Q^{-1}(u)) f_R(r) dr + \int_{d_0}^R f_R(r) dr \quad (33)$$

$$= \frac{d_0^2}{R^2} \exp\left(-\frac{P_0}{2}\right) \exp(\sqrt{P_0} Q^{-1}(u)) + \left(1 - \frac{d_0^2}{R^2}\right) \quad (34)$$

where $f_R(r) = \frac{2r}{R^2}$ for $0 \leq r \leq R$ has been used.

This gives the marginal pdf of the reported p-values v as

$$f_V(v) = \begin{cases} f_U(v) & \text{if } i \text{ is an honest sensor} \\ f_U(1-v) & \text{if } i \text{ is a Byzantine sensor} \end{cases} \quad (35)$$

Under the assumption of independent p-values of the sensor observations, the reported p-values v are also independent. The FDR control algorithm requires ordering of these reported p-values denoted by $v_{1,N} \leq v_{2,N} \leq \dots \leq v_{N,N}$ and are correlated (due to the ordering) with joint pdf given by

$$f_{V_{1,N} V_{2,N} \dots V_{N,N}} = N! f_{V_1}(v_1) f_{V_2}(v_2) \dots f_{V_N}(v_N) \quad 0 \leq v_{1,N} \leq v_{2,N} \leq \dots \leq v_{N,N} \quad (36)$$

where the marginal density $f_{V_i}(v_i)$ is given by (35)

These ordered reported p-values are compared against linearly decreasing thresholds to get the number of detections and, therefore, the probability can be found as

$$P(\Delta = i; G_1) = \int_{v_{N,N}=\gamma}^1 \dots \int_{v_{i+1,N}=\gamma}^{v_{i+2,N}} \int_{v_{i,N}=0}^{i\gamma/N} \dots \int_{v_{1,N}=0}^{v_{2,N}} N! f_{V_1}(v_1) f_{V_2}(v_2) \dots f_{V_N}(v_N) dv_{1,N} dv_{2,N} \dots dv_{N,N} \quad (37)$$

However, since any of the M sensors can be Byzantines, we need to take an average over the $\binom{N}{M}$ possibilities which gives the desired result.

For a large number of sensors, we can derive the approximate distribution of the count statistic Δ under G_1 using the result by Genovese and Wasserman which states that asymptotically the Benjamini-Hochberg method corresponds to classifying as H_1 all p-values that are less than a particular threshold v^* , where v^* is the solution to the equation

$$F(v) = \beta v \quad (38)$$

and

$$\beta = \frac{\frac{1}{\gamma} - A_0}{1 - A_0} \quad (39)$$

Here $F(v)$ is the c.d.f of the reported p-values under H_1 and is assumed to be strictly concave, and A_0 is the fraction of true H_0 s. This threshold v^* is found by assuming a mixture model of the distribution of p-values. For honest sensors, $F_H(v) = Q(Q^{-1}(v) - \phi)$ and for Byzantine sensors $F_B(v) = 1 - Q(Q^{-1}(1 - v) - \phi)$, where $\phi = \sqrt{(P_0)}$. So, under the mixture model, we have $F(v) = \alpha F_B(v) + (1 - \alpha) F_H(v)$. For a large ROI, on an average d_0^2/R^2 fractions of sensors receive the signal, and therefore $A_0 = 1 - d_0^2/R^2$.

Hence, the average probability of detection of an honest sensor is given by

$$\bar{P}_D^H = (1 - A_0)P(V < v^*|H_1) + A_0P(V < v^*|H_0) \quad (40)$$

$$= (1 - A_0) \int_0^{v^*} f_\phi(u) du + A_0 v^* \quad (41)$$

where $f_\phi(u)$ is given by (15).

Similarly, the probability of detection of a Byzantine sensor is given by

$$\bar{P}_D^B = (1 - A_0)P(V < v^*|H_1) + A_0P(V < v^*|H_0) \quad (42)$$

$$= (1 - A_0)P(1 - U < v^*|H_1) + A_0P(1 - U < v^*|H_0) \quad (43)$$

$$= (1 - A_0) \int_{v^*}^1 f_\phi(u) du + A_0 v^* \quad (44)$$

The probability of $\Delta = i$ detections (count) when the target is present is provided by

$$P(\Delta = i; G_1) = \sum_{k=\max(0, M-N+i)}^{\min(M, i)} \binom{M}{k} \binom{N-M}{i-k} (\bar{P}_D^B)^k (1 - \bar{P}_D^B)^{M-k} (\bar{P}_D^H)^{i-k} (1 - \bar{P}_D^H)^{(N-M)-(i-k)} \quad (45)$$

or

$$P(\Delta = i; G_1) = \binom{N}{i} (\bar{P}_D)^i (1 - \bar{P}_D)^{N-i} \quad (46)$$

where \bar{p}_D is the average probability of detection of a sensor given by

$$\bar{p}_D = \alpha \bar{p}_D^B + (1 - \alpha) \bar{p}_D^H \quad (47)$$

Also, we can further approximate this expression using DeMoivre-Laplace theorem as a Gaussian distribution

$$P(\Delta = i; G_1) \approx \frac{1}{\sqrt{2\pi N \bar{p}_D (1 - \bar{p}_D)}} \exp\left(-\frac{(i - N \bar{p}_D)^2}{2N \bar{p}_D (1 - \bar{p}_D)}\right) \quad (48)$$

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